arXiv:0902.1140v2 [hep-ph] 9 Jul 2009

July 2009

# $T^{'}$ Predictions of PMNS and CKM Angles

**Paul H. Frampton*** **and Shinya Matsuzaki†**

*Department of Physics and Astronomy, University of North Carolina,*
*Chapel Hill, NC 27599-3255.*

**Abstract**

Generalizing a previous model to accommodate the third quark family and CP violation, we present a $T^{'}$ model which predicts tribimaximal neutrino (PMNS) mixings while the central predictions for quark mixings are $|V_{td}/V_{ts}| = 0.245$ and $|V_{ub}/V_{cb}| = 0.237$ with a predicted CP violating KM phase $\delta_{KM} = 65.8^0$. All these are acceptably close to experiment, including the KM phase for which the allowed values are $63^0 < \delta_{KM} < 72^0$, and depend only on use of symmetry $T^{'} \times Z_2$ to define the model and no additional parameters.

*frampton@physics.unc.edu
†synya@physics.unc.edu

The justly celebrated standard model (SM) of particle phenomenology rests as one of the great accomplishments of theory. Nevertheless, in the extended form which can accommodate neutrino masses, it has 28 free parameters none of which has been calculated or predicted. Notable are quark and lepton masses (12, hereafter SM masses) and mixing angles and phases (10, hereafter SM mixings) which relate mass eigenstates for quarks and leptons to states in the lagrangian.

Except for a hint about the neutrino hierarchy, we find no new insight about SM masses [#3]. With regard to SM mixings we report at least limited progress towards understanding 6 out of 10 phenomenological parameters by the study of $T^{'}$ (binary tetrahedral group) as a flavor symmetry commuting with the SM theory. If even one of these explanations survives we feel it will be useful and important.

In recent work, the present authors, together with Kephart [1], presented a simplified model based on $T^{'}$ flavor symmetry. The principal simplification was that the CKM mixing angles [#4] involving the third quark family were taken to vanish $\Theta_{23} = \Theta_{13} = 0$.

In terms of the scalar field content, all scalar fields are taken to be doublets under electroweak $SU(2)$ with vacuum values which underly the symmetry breaking. Great simplification was originally achieved by the device of restricting scalar fields to irreducible representations of $T^{'}$ which are singlets and triplets only, without any $T^{'}$ doublets. There was a good reason for this because the admission of $T^{'}$-doublet scalars enormously complicates the symmetry breaking. This enabled the isolation of the Cabibbo angle $\Theta_{12}$ and to a reasonable prediction thereof, namely [1] $\tan 2\Theta_{12} = (\sqrt{2})/3$.

Within the same simplified model, in a subsequent paper with Eby [2], the departure of $\Theta_{12}$ from this $T^{'}$ prediction was used to make predictions for the departure of the neutrino PMNS angles $\theta_{ij}$ from their tribimaximal values [3]. Also in that model [4], we suggested a smoking-gun $T^{'}$ prediction for leptonic decay of the standard model Higgs scalar. Other related works are [5–15].

In the present article, we examine the addition of $T^{'}$-doublet scalars. As anticipated in [1], this allows more possibilities of $T^{'}$ symmetry breaking and permits non-zero values for $\Theta_{23}$, $\Theta_{13}$ and $\delta_{KM}$. We present an explicit

[#3] The result in an earlier paper [1] involving the quark mass squared ratio $(m_d^2/m_s^2)$ has only small corrections from the considerations of the present Letter.

[#4] Note that here upper case $\Theta_{ij}$ refer to quarks (CKM) and lower case $\theta_{ij}$ will refer to neutrinos (PMNS).

$(T' \times Z_2)$ model which leads to consistent results for PMNS and KM angles as well as testable predictions.

To understand the incorporation of $T'$-doublet scalars and to make the present article self-contained, it is necessary to review the simplified model employed in [1,2,4] in which $T'$-doublet scalars were deliberately excluded in order to isolate the Cabibbo angle $\Theta_{12}$. We here adopt the global symmetry $(T' \times Z_2)$.

Left-handed quark doublets $(t,b)_L, (c,d)_L, (u,d)_L$ are assigned under this as

$$\begin{array}{lll} \begin{pmatrix} t \\ b \end{pmatrix}_L & \mathcal{Q}_L & (\mathbf{1_1}, +1) \\ \left.\begin{array}{l} \begin{pmatrix} c \\ s \end{pmatrix}_L \\ \begin{pmatrix} u \\ d \end{pmatrix}_L \end{array}\right\} & Q_L & (\mathbf{2_1}, +1), \end{array} \tag{1}$$

and the six right-handed quarks as

$$\begin{array}{lll} t_R & & (\mathbf{1_1}, +1) \\ b_R & & (\mathbf{1_2}, -1) \\ \left.\begin{array}{l} c_R \\ u_R \end{array}\right\} & \mathcal{C}_R & (\mathbf{2_3}, -1) \\ \left.\begin{array}{l} s_R \\ d_R \end{array}\right\} & \mathcal{S}_R & (\mathbf{2_2}, +1). \end{array} \tag{2}$$

The leptons are assigned as

$$\left.\begin{array}{l} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \end{array}\right\} L_L(3,+1) \quad \begin{array}{ll} \tau_R^- \ (1_1,-1) & N_R^{(1)} \ (1_1,+1) \\ \mu_R^- \ (1_2,-1) & N_R^{(2)} \ (1_2,+1) \\ e_R^- \ (1_3,-1) & N_R^{(3)} \ (1_3,+1), \end{array} \tag{3}$$

Next we turn to the symmetry breaking and the necessary scalar sector with its own potential [#5] and Yukawa coupling to the fermions, leptons and quarks.

[#5] The scalar potential will not be examined explicitly. We assume that it has enough parameters to accommodate the required VEVs in a finite neighborhood of parameter values.

For scalar fields we keep those used previously namely the two $T^{'}$ triplets and two $T^{'}$ singlets

$$H_3(3,+1);\ H_3^{'}(3,-1);\ H_{1_1}(1_1,+1);\ H_{1_3}(1_3,-1) \tag{4}$$

which led to the simplified model discussed previously with CKM angles $\Theta_{23} = \Theta_{13} = 0$. That model was used to derive a formula for the Cabibbo angle [1], to predict corrections [2] to the tribimaximal values [3] of PMNS neutrino angles, and to make a prediction for Higgs boson decay [4].

We now introduce one $T^{'}$ doublet scalar in an explicit model. Non-vanishing $\Theta_{23}$ and $\Theta_{13}$ will be induced by symmetry breaking due to the addition the $T^{'}$ doublet scalar

$$H_{2_3}(2_3,+1) \tag{5}$$

The field in Eq.(5) allows a new [#6] Yukawa coupling

$$Y_{\mathcal{QS}}\mathcal{Q}_L\mathcal{S}_R H_{2_3} + h.c. \tag{6}$$

where $Y_{sb} = Y_{sb}^*$ is real and we accommodate CP violation by the use of complex $T^{'}$ Clebsch-Gordan coefficients.

The vacuum expectation value (VEV) for $H_{2_3}$ is taken with the alignment

$$< H_{2_3} >= V_{2_3}(1,1) \tag{7}$$

[#6] We list the Yukawa couplings already discussed for the $T^{'}$-triplet and $T^{'}$-singlet scalars:

$$\begin{aligned}
\mathcal{L}_Y \; = \; & \frac{1}{2}M_1 N_R^{(1)}N_R^{(1)} + M_{23}N_R^{(2)}N_R^{(3)} \\
& + \left\{ Y_1\left(L_L N_R^{(1)} H_3\right) + Y_2\left(L_L N_R^{(2)} H_3\right) + Y_3\left(L_L N_R^{(3)} H_3\right)\right. \\
& \left. + Y_\tau\left(L_L \tau_R H_3^{\prime}\right) + Y_\mu\left(L_L \mu_R H_3^{\prime}\right) + Y_e\left(L_L e_R H_3^{\prime}\right)\right\} \\
& + Y_t(\{\mathcal{Q}_L\}_{\mathbf{1_1}}\{t_R\}_{\mathbf{1_1}}H_{\mathbf{1_1}}) \\
& + Y_b(\{\mathcal{Q}_L\}_{\mathbf{1_1}}\{b_R\}_{\mathbf{1_2}}H_{\mathbf{1_3}}) \\
& + Y_{\mathcal{C}}(\{Q_L\}_{\mathbf{2_1}}\{\mathcal{C}_R\}_{\mathbf{2_3}}H_{\mathbf{3}}^{'}) \\
& + Y_{\mathcal{S}}(\{Q_L\}_{\mathbf{2_1}}\{\mathcal{S}_R\}_{\mathbf{2_2}}H_{\mathbf{3}}) \\
& + \text{h.c.}.
\end{aligned}$$

while as in [1] the other VEVs include

$$< H_3 >= V(1, -2, 1) \tag{8}$$

The hermitian squared mass matrix $\mathcal{D} \equiv DD^\dagger$ for the charge $(-1/3)$ quarks is then

$$\mathcal{D} = \begin{pmatrix} m_b^2 & \frac{1}{\sqrt{6}} Y_{\mathcal{S}} Y_{\mathcal{QS}} V V_{2_3} (1 - 2\sqrt{2}\omega^2) & \frac{1}{\sqrt{6}} Y_{\mathcal{S}} Y_{\mathcal{QS}} V V_{2_3} (\omega^2 + \sqrt{2}) \\ \frac{1}{\sqrt{6}} Y_{\mathcal{S}} Y_{\mathcal{QS}} V V_{2_3} (1 - 2\sqrt{2}\omega^{-2}) & 3(Y_{\mathcal{S}} V)^2 & -\frac{\sqrt{2}}{3}(Y_{\mathcal{S}} V)^2 \\ \frac{1}{\sqrt{6}} Y_{\mathcal{S}} Y_{\mathcal{QS}} V V_{2_3} (\omega^{-2} + \sqrt{2}) & -\frac{\sqrt{2}}{3}(Y_{\mathcal{S}} V)^2 & (Y_{\mathcal{S}} V)^2 \end{pmatrix} \tag{9}$$

In Eq.(9) the $2 \times 2$ sub-matrix for the first two families coincides with the result discussed earlier [1] and hence the successful Cabibbo angle formula $\tan 2\Theta_{12} = (\sqrt{3})/2$ is preserved.

Note that in this model the mass matrix for the charge $+2/3$ quarks is diagonal [#7] so the CKM mixing matrix arises purely from diagonalization of $\mathcal{D}$ in Eq.(9). The presence of the complex $T^{'}$ Clebsch-Gordan coefficients [16] in Eq.(9) which will lead to non-zero KM CP violating phase.

For $m_b^2$ the experimental value $17.6 GeV^2$ [17] although the CKM angles and phase do not depend on this overall normalization.

Actually our results depend only on assuming that the ratio $(Y_{\mathcal{QS}} V_{2_3} / Y_{\mathcal{S}} V)$ is much smaller than one.

In this case we find the results for the ratios of CKM matrix elements

$$|V_{td}/V_{ts}| = 0.245 \tag{10}$$

and

$$|V_{ub}/V_{cb}| = 0.237 \tag{11}$$

For the CP violating phase $\delta_{KM}$

$$\delta_{KM} = 65.8^0 \tag{12}$$

is in the experimentally allowed range $63^0 < \delta_{KM} < 72^0$.

[#7] This uses the approximation that the electron mass is $m_e = 0$; *c.f.* ref. [1].

More technical details will be provided in [18].

Note that once the off-diagonal third-family elements in Eq.(9) are taken as much smaller than the elements involved in the Cabibbo angle, the two KM angles and the CP phase are predicted without further free parameters so this vindicates the hope expressed in [1].

In summary, we have reported progress in understanding the PMNS and CKM mixing angles by exploiting the binary tetrahedral group ($T^{'}$) as a global discrete flavor symmetry commuting with the local gauge symmetry $SU(3) \times SU(2) \times U(1)$ of the standard model of particle phenomenology. The results are encouraging to pursue this direction of study.

# Acknowledgements

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-06ER41418.

# References


[1] P.H. Frampton, T.W. Kephart and S. Matsuzaki. Phys. Rev. **D78,** 073004 (2008). `arXiv:0807.4713 [hep-ph]`.

[2] D.A. Eby, P.H. Frampton and S. Matsuzaki, Phys. Lett. **B671,** 386 (2009). `arXiv:0810.4899 [hep-ph]`.

[3] P.F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) `hep-ph/0202074`.

[4] P.H. Frampton and S. Matsuzaki, Mod. Phys. Lett. **A** (2009, in press). `arXiv:0810.1029 [hep-ph]`.

[5] P.H. Frampton and S. Matsuzaki, Mod. Phys. Lett. **A24,** 429 (2009). `arXiv:0807.4785 [hep-ph]`;

[6] E. Ma and G. Rajasekaran, Phys. Rev. **D64,** 113012 (2001). `hep-ph/0106291`;
K.S. Babu, E. Ma and J.W.F. Valle, Phys. Lett. **B552,** 207 (2003). `hep-ph/0206292`.

[7] E. Ma, Mod. Phys. Lett. **A20,** 2601 (2005). `hep-ph/0508099`; Phys. Lett. **B632,** 352 (2006). `hep-ph/0508231`.
B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M.K. Parida, Phys. Lett. **B638,** 345 (2006). `hep-ph/0603059`.
E. Ma, Phys. Rev. **D73,** 057304 (2006).
E. Ma, H. Sawanaka, and M. Tanimoto, Phys. Lett. **B641,** 301 (2006). `hep-ph/0606103`.
E. Ma, Mod. Phys. Lett. **A21,** 1917 (2006). `hep-ph/0607056`.

[8] G. Altarelli and F. Feruglio, Nucl. Phys. **B720,** 64 (2005). `hep-ph/0504165`; Nucl. Phys. **B742,** 215 (2006). `hep-ph/0512103`;
G. Altarelli, F. Feruglio and Y. Lin, Nucl. Phys. **B775,** 31 (2007). `hep-ph/0610165`;

[9] P.H. Frampton and T.W. Kephart, Int. J. Mod. Phys. **10A,** 4689 (1995); JHEP 09:110 (2007). `arXiv:0706.1186 [hep-ph]`.

[10] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. **B775,** 120 (2007). `hep-ph/0702194`.

[11] M.-C. Chen and K.T. Mahanthappa. Phys. Lett. **B652,** 34 (2007). `arXiv:0705.0714`.

[12] P.F. Harrison and W.G. Scott, Phys. Lett. **B628,** 93 (2005). hep-ph/0508012.

[13] P.H. Frampton and S.L. Glashow, Phys. Lett. **B461,** 95 (1999). hep-ph/9906375

[14] P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. **536B,** 79 (2002). hep-ph/0201008.

[15] P.H. Frampton, S.L. Glashow and T. Yanagida, Phys. Lett. **B548,** 119 (2002). hep-ph/0208157.

[16] M.-C. Chen and K.T. Mahanthappa, arXiv:0904.1721 [hep-ph]

[17] Particle Data Group, Phys. Lett. **667,** 1 (2008).

[18] D.A. Eby, P.H. Frampton and S. Matsuzaki, in preparation.